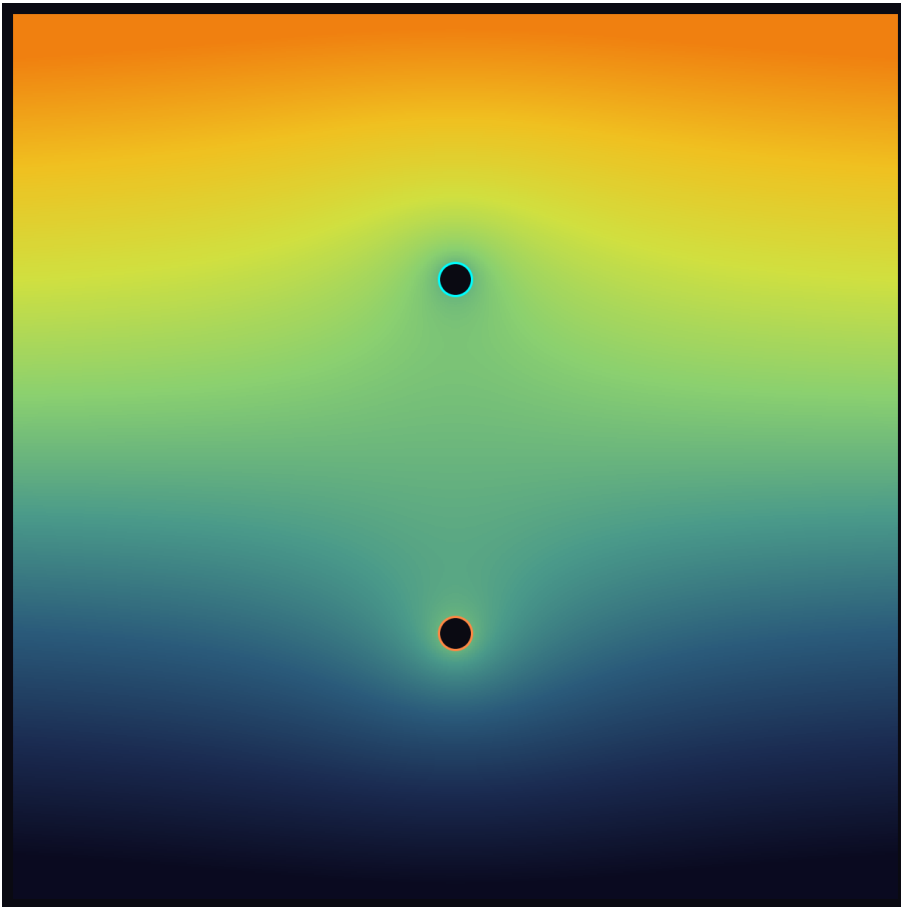


# Gravitational potential in worlds with portals

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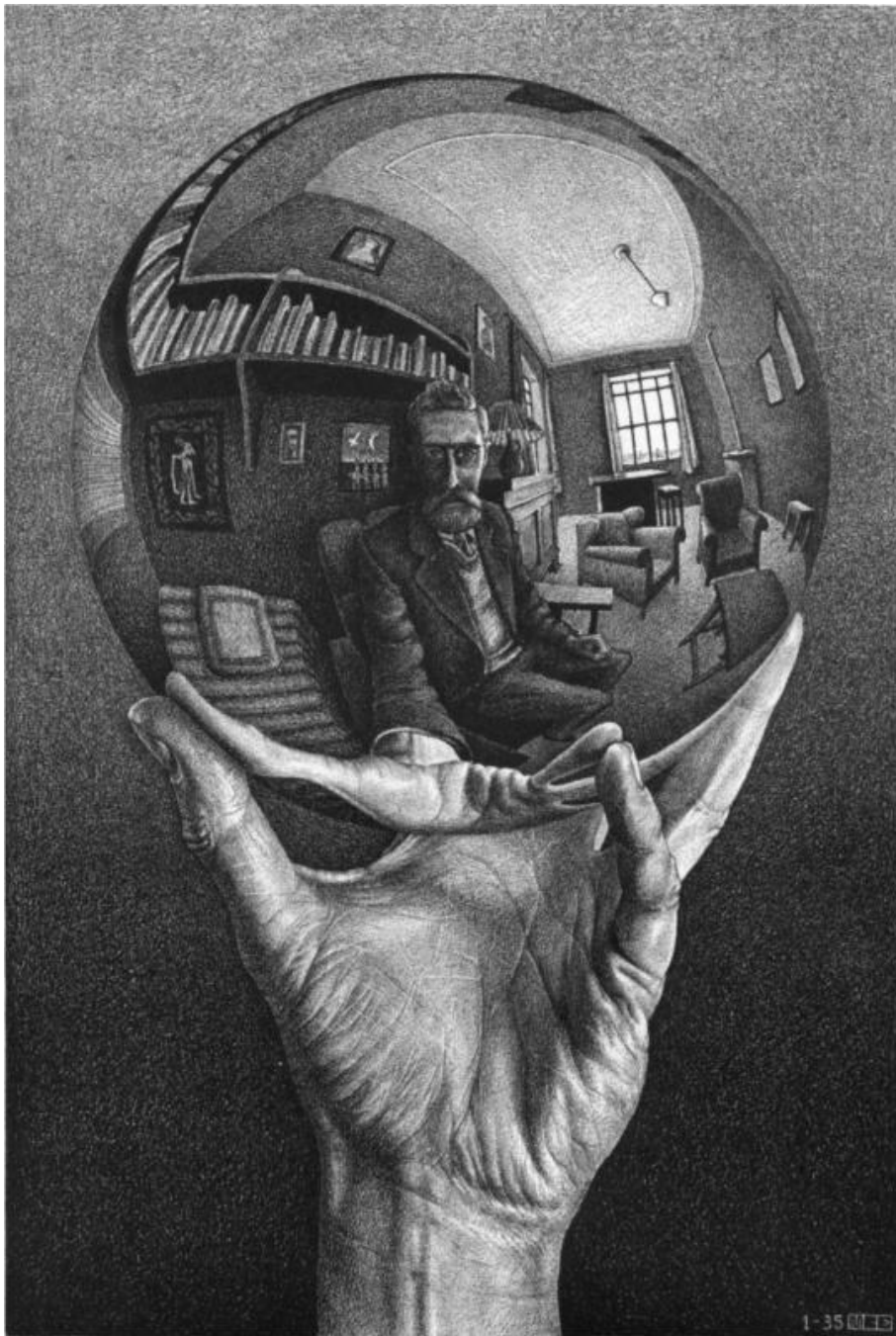


Figure 1: Escher, M.C. Hand with Reflecting Sphere (Self-Portrait in Spherical Mirror). 1935. Lithograph, 31.8 × 21.3 cm.

## Quick reference

**Mundane space** In two dimensions, a point mass creates a gravitational potential of

$$\Phi(\mathbf{r}) = -m \log |\mathbf{r} - \mathbf{r}_0|.$$

Here,  $\mathbf{r}_0$  is the location of the point mass,  $m$  is its mass, and  $\mathbf{r}$  is any point in the universe.

**Narnian portals** In two dimensions with a narnian portal between universes A and B, a point mass in universe A creates a gravitational potential:

$$\begin{aligned}\Phi^A(\mathbf{r}) &= -m \log |\mathbf{r} - \mathbf{r}_0| + \frac{1}{2} m \log |\mathbf{r}| \\ \Phi^B(\mathbf{r}) &= -m \log \left| \mathbf{r} + \frac{a^2}{\mathbf{r}_0} \right| + \frac{1}{2} m \log |\mathbf{r}|\end{aligned}$$

Here,  $\mathbf{r}_0$  is the location of the point mass,  $m$  is its mass,  $a$  is the radius of the circular portal at the origin,  $\Phi^A$  and  $\Phi^B$  are the potentials throughout universes A and B respectively, and  $\mathbf{r}$  is any point in that universe.

**Gladosian portals** In two dimensions with a gladosian portal, a point mass creates a gravitational potential:

$$\begin{aligned}\Phi(\tau, \sigma) &= -m \sum_{n=-\infty}^{\infty} \log(\cosh(\tau - \tau_1 - 2n\tau_0) - \cos(\sigma - \sigma_1)) \\ &\quad + m \sum_{n \neq 0} \log(\cosh(\tau - 2n\tau_0) - \cos \sigma)\end{aligned}$$

Here,  $\langle \tau, \sigma \rangle$  is the location of a point specified in bipolar coordinates,  $\langle \tau_1, \sigma_1 \rangle$  is the location of the point mass and  $m$  is its mass, and  $\tau_0$  is the radius of the two circular portals expressed as a contour of constant  $\tau = \pm\tau_0$ .

**Near a planetary surface** Near a planet's surface, with a pair of stacked gladosian portals at different altitudes, the gravitational potential becomes:

$$\Phi(\tau, \sigma) = \frac{c \sinh \tau}{\cosh \tau - \cos \sigma} - \frac{c}{\tau_0} \tau - 2c \sum_{n=1}^{\infty} \frac{e^{-n\tau_0}}{\sinh(n\tau_0)} \sinh(n\tau) \cos(n\sigma)$$

Here, the first term is the usual gravitational potential that varies linearly with altitude, and the rest is a harmonic correction to account for portal geometry. The straight-line separation between the foci<sup>1</sup> is  $2c$ .

Fascinatingly, the rims of both portals are isopotential surfaces  $\Phi(\pm\tau_0, \sigma) = 0$  all the way around.

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<sup>1</sup>I forget if it's the foci or the portal centers; they're slightly displaced from one another.

# 1 Narnian portals

Imagine you open up a portal between two universes, allowing light and matter to travel between them. The portal behaves like a window, because photons from the other universe travel through it and impinge on your eyes. And it behaves like a door, because matter can traverse the portal, too.

What about gravity? If you have a massive object on the other side of the portal, what kind of gravitational force does it exert on this side? The aim of this article is to derive the unique law of Newtonian gravity for conjoined universes with a portal between them.

In short, the results are as follows:

1. **Gravity flows between universes.** Objects on one side are indeed attracted into the portal by massive objects on the other side.
2. **Portals decrease apparent mass.** Viewed from very far away, an object of mass  $M$  in a world-with-a-portal exerts the same gravitational field that an object of mass  $M/2$  ordinarily would have. This  $M/2$  field is the same on both sides of the portal; it is exactly as if the portal siphons gravitational flux so that each side gets half, which makes objects seem gravitationally half as powerful.
3. **Gravity behaves normally up close.** Up close, an object of mass  $M$  exerts the expected amount of gravitational force. It doesn't matter whether you're in a world with a narnian portal or not; you can't tell up close.
4. **Objects are attracted toward portal images.** It is like holding a mirrored ball in your hand and seeing the whole world captured in a spherical image (Figure 1): the portal appears to contain the entirety of the neighboring universe just under its surface. When there's a mass in the other universe, objects in this universe are attracted toward the specific image point under the surface of the portal where the mass *seems* to be.

Formally, the equation governing gravity for narnian portals is:

$$\begin{aligned}\Phi^A(\mathbf{r}) &= \log|\mathbf{r} - \mathbf{r}_0| - \frac{1}{2} \log|\mathbf{r}| \\ \Phi^B(\mathbf{r}) &= \log\left|\mathbf{r} + \frac{a^2}{\mathbf{r}_0}\right| - \frac{1}{2} \log|\mathbf{r}|\end{aligned}$$

In words: you have two universes A and B which each look like the 2D plane<sup>2</sup>. Around the origin of both planes, you cut out a circular portal of radius  $a$ , then glue the two universes together along those matching circular rims to make a portal between them.

If you put a mass at point  $\mathbf{r}_0$  in universe A, it creates a gravitational potential  $\Phi^A$  throughout its own universe, and a potential  $\Phi^B$  throughout the other universe. (Objects fall from high to low gravitational potential.)

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<sup>2</sup>These gravitational portal equations are for 2D flatland; they look different and more complicated in 3D.

In its own universe, the mass's gravitational potential consists of the  $\log|\mathbf{r} - \mathbf{r}_0|$  term—which is the ordinary 2D gravitational attraction pulling an object at position  $\mathbf{r}$  toward the mass at position  $\mathbf{r}_0$ —minus a  $\frac{1}{2}\log|\mathbf{r}|$  term, which is the attenuation of the field due to the portal at the origin. Up close to the mass, the first term dominates and so the potential looks normal, like  $\log|\mathbf{r} - \mathbf{r}_0|$ . Very far away,  $\mathbf{r}$  becomes large enough that  $\mathbf{r} - \mathbf{r}_0 \approx \mathbf{r}$ , and so the potential looks like  $\frac{1}{2}\log|\mathbf{r} - \mathbf{r}_0|$ —half of its typical value.

In the neighboring universe B, there are also two forces at play: objects are attracted into the portal by the mass in universe A, but the attractive force is attenuated by a countervailing force from the portal at the origin. The specific point  $-\frac{a^2}{\mathbf{r}_0}$  is the “portal image” of the mass: when you gaze into the portal in universe B, the mass from universe A appears to be just below the surface of the portal at this point. Objects throughout universe B are attracted into the portal just as if the mass were actually stationed below the surface there.

## The mathematics of narnian portals

In the sections that follow, we'll define the formalism and prove the gravitational law I just described.

**Two planes with a hole in the middle** We start with the ordinary Euclidean plane and remove a disc of radius  $a$  centered at the origin, forming a space we'll call  $\mathcal{P}$ . By gluing two copies of  $\mathcal{P}$  along the disc boundary in an appropriate way, you create a portal between the two universes such that entering the disc in one universe causes you to exit the disc in the other universe while keeping the same heading. In polar coordinates, the appropriate corresponding points to glue together are  $\langle a, \theta \rangle_1 \sim \langle a, \theta + \pi \rangle_2$  for all  $\theta$ .

**Gravitational potential** An object with mass creates a gravitational potential throughout all of space. Gravitational potential is a number  $\phi(\mathbf{r})$  assigned to each point  $\mathbf{r}$  in space, where gravitational attraction causes objects to fall from high to low potential. Gravitational potentials superimpose, so you can determine the combined gravitational effect of two objects by adding their gravitational potential fields together.

When we say we want to find out how gravity works in a universe with portals, what we mean is we want to know what gravitational potential field objects create when they live in a world with portals.

**You can find gravity by solving Poisson's equation** There is a differential equation you can solve in order to determine the gravitational potential in any space. If you put an idealized object of mass  $m$  at a single point  $\mathbf{r}_0$ , it creates a potential field  $\phi(\mathbf{r})$  at every point in space, and this field must obey Poisson's equation:

$$\nabla^2 \phi(\mathbf{r}) = m\delta(\mathbf{r} - \mathbf{r}_0)$$

Here,  $\nabla^2\phi$  is shorthand for the second derivatives  $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}$ , and  $\delta(\mathbf{r} - \mathbf{r}_0)$  is the dirac delta function, which is zero at every point  $\mathbf{r}$  except  $\mathbf{r} = \mathbf{r}_0$ .

As you can tell from the derivatives, Poisson's equation is a second-order differential equation. By solving this equation for the unknown function  $\phi(\mathbf{r})$ , we can find out what potential an object of mass  $m$  and position  $\mathbf{r}_0$  creates in this portal space. (And because potential fields add together, this tells you everything you need to know about how gravity behaves in portal space no matter what arrangement of masses you have.)

**Ordinarily, gravitational potential behaves like  $\log|\mathbf{r} - \mathbf{r}_0|$**  If you solve Poisson's equation in the ordinary plane, you get the basic 2D gravitational potential:

$$\phi_0(\mathbf{r}) = \log|\mathbf{r} - \mathbf{r}_0|$$

We'll keep this in mind as a template solution for our portal world. Although the potential in portal world may be different, knowing the ordinary 2D solution may be useful as a starting point.

**Three boundary conditions** Here are three common-sense requirements that I think the gravitational potential ought to satisfy in our portal space:

1. Corresponding points on the portals must have the same potential. (After all, they're glued together, so they should match.)
2. Corresponding points on the portals must have the same radial flux  $\partial\phi/\partial r$ . (Potential should change smoothly as you cross the seam)
3. The potential should change smoothly when a massive object goes through the portal. (The gravitational potential shouldn't suddenly snap to a new value as a new object crosses over.)

These requirements are *boundary conditions*; in our case, they'll help us find solutions to Poisson's equation that respect the topology of our space, i.e. that take into account the presence of the portal.

**Potentials for two universes** We've placed a point mass  $m$  at location  $\mathbf{r}_0$  in our first universe. This creates a potential field we'll denote by  $\phi^A$  throughout this universe A, and a potential field  $\phi^B$  throughout the neighboring universe B. Our job is to determine the unknown potentials  $\phi^A$  and  $\phi^B$  by solving Poisson's equation.

In universe A, Poisson's equation is:

$$\nabla^2\phi^A(\mathbf{r}) = m\delta(\mathbf{r} - \mathbf{r}_0)$$

In universe B, there isn't any mass at all, so the equation is just

$$\nabla^2\phi^B(\mathbf{r}) = 0$$

Remember that  $\delta(\mathbf{r} - \mathbf{r}_0)$  is zero everywhere except the single point where the mass is, so these equations are not so different—they're actually exactly the same but for that one point  $\mathbf{r}_0$  in universe A.

**We can build the potential out of harmonic building blocks** What kinds of functions  $f$  satisfy the differential equation  $\nabla^2 f = 0$  anyways?

In polar coordinates, the function  $f(r, \theta) = \log(r)$  satisfies this differential equation. So do the functions  $f(r, \theta) = r^n \cos(n\theta)$  and  $f(r, \theta) = r^n \sin(n\theta)$  for any integer  $n$ .

And in fact, according to fourier analysis, these so-called harmonic functions are the building blocks for *any* solution to the equation  $\nabla^2 f = 0$ . In particular:

1. Every well-behaved function that satisfies the differential equation  $\nabla^2 f = 0$  can be written as a weighted sum of these harmonic functions.
2. The weights are unique—if you can build a function  $f$  as a weighted sum of harmonic functions, there's exactly one set of coefficients that can do it.

So, assuming optimistically that our potentials are well-behaved, we can write them as a weighted combination of harmonic functions with unknown weights. And by applying our common-sense boundary conditions, we can solve for the weights and determine the potentials exactly.

#### A sketch of the solution

1. Assume the solution has the form  $\phi^A(\mathbf{r}) = \phi_0(\mathbf{r}) + \psi^A(\mathbf{r})$  and  $\phi^B(\mathbf{r}) = \psi^B(\mathbf{r})$ , where  $\phi_0$  is the gravitational potential of a point mass in ordinary 2D space, and  $\psi(\mathbf{r})$  is a harmonic correction factor to make sure that  $\phi$  satisfies the boundary conditions (shape requirements) of our space.
2. Assuming  $\psi(\mathbf{r})$  is the most general harmonic function, our potential looks like:

$$\psi^A(\mathbf{r}) = A \cdot \log|\mathbf{r} - \mathbf{r}_0| + C \log(|\mathbf{r}|) + \sum_{n=-\infty}^{\infty} A_n r^n \cos(n\theta) + \sum_{n=-\infty}^{\infty} B_n r^n \sin(n\theta),$$

$$\psi^B(\mathbf{r}) = D \log(|\mathbf{r}|) + \sum_{n=-\infty}^{\infty} E_n r^n \cos(n\theta) + \sum_{n=-\infty}^{\infty} F_n r^n \sin(n\theta),$$

where all the  $A_i$  and  $B_i$  and  $C$  and  $D$  and  $E_i$  and  $F_i$  are constants we need to determine.

3. Because of Gauss's law, the potential has to look like  $\phi_0$  from far away. This means that all the  $r^n$  terms in our correction must have a negative exponent so they go to zero.
4. Take the fourier transform of  $\log(|\mathbf{r} - \mathbf{r}_0|)$  to express it in terms of harmonic functions. Then apply the first boundary condition and match up the coefficients. This gives you equations relating the coefficients to one another.

5. Take the derivative of  $\phi$  in the normal (i.e. radial  $\hat{r}$ ) direction. Then apply the second boundary condition and match up the coefficients. This gives you further equations relating the coefficients. You can now solve for almost all of them.
6. The remaining major unknown is a factor determining how much flux is shared between the two universes. Requiring the third boundary condition shows that the flux is shared symmetrically, half and half.
7. We find

$$\begin{aligned}\Phi^A(\mathbf{r}) &= -m \log|\mathbf{r} - \mathbf{r}_0| + \frac{1}{2}m \log|\mathbf{r}| \\ \Phi^B(\mathbf{r}) &= -m \log\left|\mathbf{r} + \frac{\mathbf{a}^2}{\mathbf{r}_0}\right| + \frac{1}{2}m \log|\mathbf{r}|\end{aligned}$$

## 2 Gladosian portals

Imagine you create a portal joining two parts of your universe together so you can step between faraway places in an instant. There are some important questions about how gravity behaves here:

1. What happens if there's a heavy object like a planet on the other side of the portal—does it gravitationally draw you in?
2. Can you create endless acceleration by putting one portal high up and the other low down and falling between them forever?
3. If you arrange the portals so that you can see yourself, do you exert gravitational attraction on *yourself* through the portal?

And the answers are:

1. **Gravity flows between universes.** Yes, objects on opposite sides of the portal gravitationally attract one another.
2. Like two mirrors facing one another, portals create nested reflections of the entire universe underneath their surface. Although there's nothing “under the surface” of a portal, gravity behaves just as if those images are real and have mass.
3. **Portals don't cheat energy.** You can't create endless acceleration with portals. When you place one high and one low, several things happen: first, they stretch the gravity field between them like taffy so that the space between them is nearly gravitationally level and does not accelerate you. Second, they pucker the space around them so that climbing into the bottom portal and out of the top portal is an uphill climb that takes just as much formal work as climbing a ladder between the two portals. The short proof is that because we've found a (single-valued) potential, the associated force *must* conserve energy.
4. **Objects don't attract themselves.** Objects are not attracted to *themselves* through portals. Because spaces with portals are curved, unlike ordinary flat space, it is possible to see your own self through a portal and



point at yourself. But the position you are pointing at isn't different from the one you're standing in. There is no force pulling you from *here* to *there* because here and there are the same location.

## The mathematics of gladosian portals

While a narnian portal joins two universes together, a gladosian portal joins one universe to itself. Take the ordinary euclidean plane and cut out two circles from it. Then glue the circular rims of the holes to each other. (If you imagine the euclidean plane as a sheet of paper with two circles cut out of it, fold the paper so that the circles overlap and then glue the holes together.)

We can solve for the gravitational potential in this space, too, using poisson's equation:

$$\nabla^2 \phi(\mathbf{r}) = m\delta(\mathbf{r} - \mathbf{r}_0)$$

### Proof sketch

1. **Use bipolar coordinates.** Bipolar coordinates (see Wikipedia)  $\langle \tau, \sigma \rangle$  are the ideal coordinate system for representing a system with two portals.
2. **Portal circles are constant contours.** You can choose your coordinate system so that the two circles are located symmetrically on the x-axis centered on  $x = \pm d$  and with radius  $a$ . In bipolar coordinates, the left circle is the contour  $\tau = -\tau_0$  and the right circle is the contour  $\tau = +\tau_0$ . This means that all of reality on the exterior of the two portals is in the range  $-\tau_0 \leq \tau \leq +\tau_0$ .
3. **The laplacian is simple in bipolar coordinates.** The equation  $\nabla^2 \phi = 0$  holds throughout all space except at the point mass. Although  $\nabla^2$  looks complicated in bipolar coordinates, the equation  $\nabla^2 \phi = 0$  is much simpler.  
In bipolar coordinates,  $\nabla^2$  is zero just when  $\frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial \sigma^2}$  is zero. (This is because bipolar coordinates are "conformal".)
4. **You mustn't glue portals together backwards.** What's the equation gluing the two portal rims together? We have a few choices, and if we glue them backwards it will warp space. The correct gluing is to make  $\langle +\tau_0, \sigma \rangle \sim \langle -\tau_0, \sigma \rangle$  for every  $\sigma$ . This makes the coordinate rectangle  $[-\tau_0, +\tau_0] \times [0, 2\pi]$  into a torus (with a hole at the origin).  
If you glue it the wrong way like I did when trying out, you do  $\langle +\tau_0, \sigma \rangle \sim \langle -\tau_0, -\sigma \rangle$  and space becomes a klein bottle. This is undesirable because if you complete a loop through both portals when they're set up like this, you flip into a mirror image version of yourself.
5. **The portal rims must have matching potential.** Unlike the narnian case, we only need two boundary conditions to respect this gluing: first, the potential at corresponding points on the portal rims must be the same. Second, their flux must be the same.

6. **A pair of portals creates infinite refracted images.** When we glue the portal rims together and make  $+\tau_0$  equivalent to  $-\tau_0$ , we make  $\tau$  equivalent to  $\tau + 2\tau_0$  and  $\tau + 4\tau_0$  and  $\tau \pm 2n\tau_0$ .

According to the method of images, we can see fictitious images of our point mass  $\langle \tau_m, \sigma_m \rangle$  at  $\langle \tau_m + 2n\tau_0, \sigma_m \rangle$  for every integer  $n$ . And gravitational potential behaves exactly as if there really are masses in those locations. Those masses seem to be “in the interior of the portal disc” where nothing actually is—we cut out those discs to make the portal—but the math behaves as if they are.

7. **The infinite sum needs to be regularized.** And so the gravitational potential  $\phi$  is an infinite sum of the gravitational contributions of all the images in all the portals. Each one contributes a potential term  $\phi_i(\mathbf{r}) = |\mathbf{r} - \mathbf{r}_0|$ . Unfortunately, the infinitely many reflections make the total potential go to infinity.

To solve this problem, we apply *regularization*. We know that you can add a constant to potential without changing its physical properties. We’ll make sure that as we take the sum of more and more reflections, we’ll subtract off an adaptive constant so that the overall potential doesn’t become infinite.

8. **Regularization creates sink charges.** The method of regularization creates “sinks” of negative mass.

I will note cryptically that positionally they are the portal reflections of the vanishing point  $\langle \tau = 0, \sigma = 0 \rangle$ . Understanding this statement isn’t necessary for the derivation.

9. **Find the harmonic building blocks.** As we did before with the fourier transform. Note that because the bipolar angle  $\sigma$  is periodic, we get terms like  $\exp(k\sigma)$ .

#### 10. Apply the boundary conditions

11. **Hyperbolic trig identities to the rescue.** The sum of a bunch of translated logarithms can be more compactly written in terms of cosh and sinh.

There are also compact ways to rewrite them in terms of the jacobi theta function  $\vartheta$ , which is a special function for capturing the behavior of systems that are periodic in two directions, like toruses or crystal lattices or, in our case,  $\langle \tau + 2m\tau_0, \sigma + 2\pi n \rangle$ . But I prefer hyperbolic trig functions personally.

12. We find

$$\begin{aligned} \Phi(\tau, \sigma) = & -m \sum_{n=-\infty}^{\infty} \log(\cosh([\tau - \tau_1] - 2n\tau_0) - \cos(\sigma - \sigma_1)) \\ & + m \sum_{n \neq 0} \log(\cosh(\tau - 2n\tau_0) - \cos \sigma) \end{aligned}$$

Here,  $\langle \tau, \sigma \rangle$  is the location of a point specified in bipolar coordinates,  $\langle \tau_1, \sigma_1 \rangle$  is the location of the point mass and  $m$  is its mass, and  $\tau_0$  is the radius of the two circular portals expressed as a contour of constant  $\tau = \pm \tau_0$ .

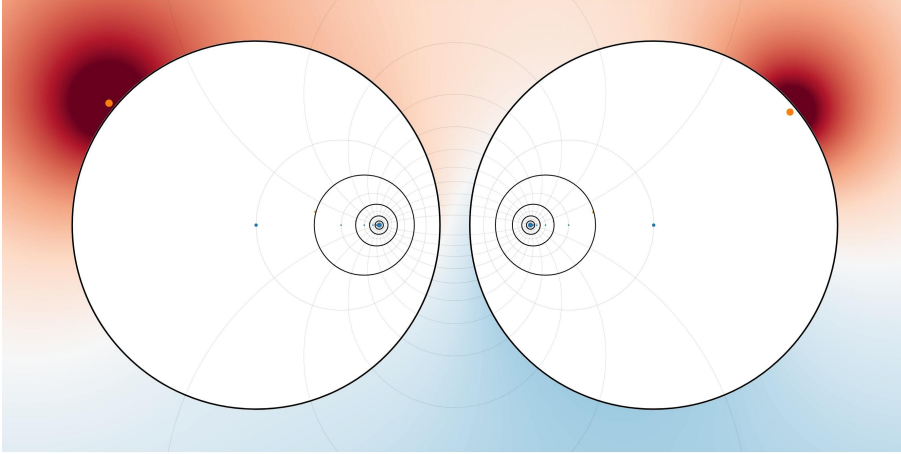


Figure 2: An orange point mass sits in the plane near a pair of gladosian portals. The gravitational potential (shown as a heat map) is warped by the presence of the portals, leaking in one and out the other. Like two mirrors facing each other, the portals create imaginary reflections of the point mass (orange) and the sinks at infinity (blue); gravitational potential behaves as if those images are real.

The reflections contain concentric circles that reflect the whole world. The outer circle represents one portal, the inner circle represents the other, and between them is all of space. In particular, you can see a reflection of the orange mass near one of the circles, where it appears in real life. And there is a blue dot acting as a matter sink; its position is a reflected vanishing point representing things infinitely far away.

## 2.1 Portals near a planetary surface

In a 2D world, if you're very close to a planet's surface, you experience gravitational potential proportional to your altitude; call it  $\phi_{\perp}(x, y) = +g \cdot y$ . If you had a portal, you could go from very low to very high in an instant. Do portals let you cheat energy conservation?

They don't. Let's set up a vertically stacked pair of portals and solve poisson's equation for the resulting gravitational potential.

If you use bipolar coordinates with the usual transformation, then they're stacked in the horizontal ( $x$ ) direction; we can either use a different conversion or declare that down is in the  $-x$  direction. Also unlike before when we set

$\tau_0$ , which controls both the size of the portals and their separation with a single number, we can write  $\tau_0$  terms of the portal radius  $a$  and the separation between foci  $2c$  and set those directly. This allows us to experiment with different degrees of portal separation  $c$ .

Using the same strategy as we did in the previous section, we'll assume that gravitational potential is equal to its usual linear-downward potential  $\phi_{\perp}(x, y) = +g \cdot x$ , plus a harmonic correction  $\phi(x, y)$  that accounts for the portal warping.

Using fourier decomposition and applying the two boundary conditions (same potential and flux around the portal rims), we find:

$$\Phi(\tau, \sigma) = \frac{c \sinh \tau}{\cosh \tau - \cos \sigma} - \frac{c}{\tau_0} \tau - 2c \sum_{n=1}^{\infty} \frac{e^{-n\tau_0}}{\sinh(n\tau_0)} \sinh(n\tau) \cos(n\sigma)$$

The first term is just  $+g \cdot x$  expressed in bipolar coordinates. The second term levels the field between the two portals to place them at the same potential and fully neutralize the effect of the gravity well. The remaining terms create a gravitational lensing effect so that close up to the portals, gravity radially inward/outward instead of down toward the planet; I believe this is what satisfies the matching-flux boundary condition.

The rotationally-symmetric gravitational puckering around both portals is fascinating. It means that it requires uphill work to approach the lower portal from any direction, and uphill work to leave the upper portal from any direction. That additional uphill work is equivalent to the amount of work it would take to cross the distance between the two portals in free space, exactly canceling the gravitational benefit of taking a portal shortcut. You can't cheat conservation of energy using portals.

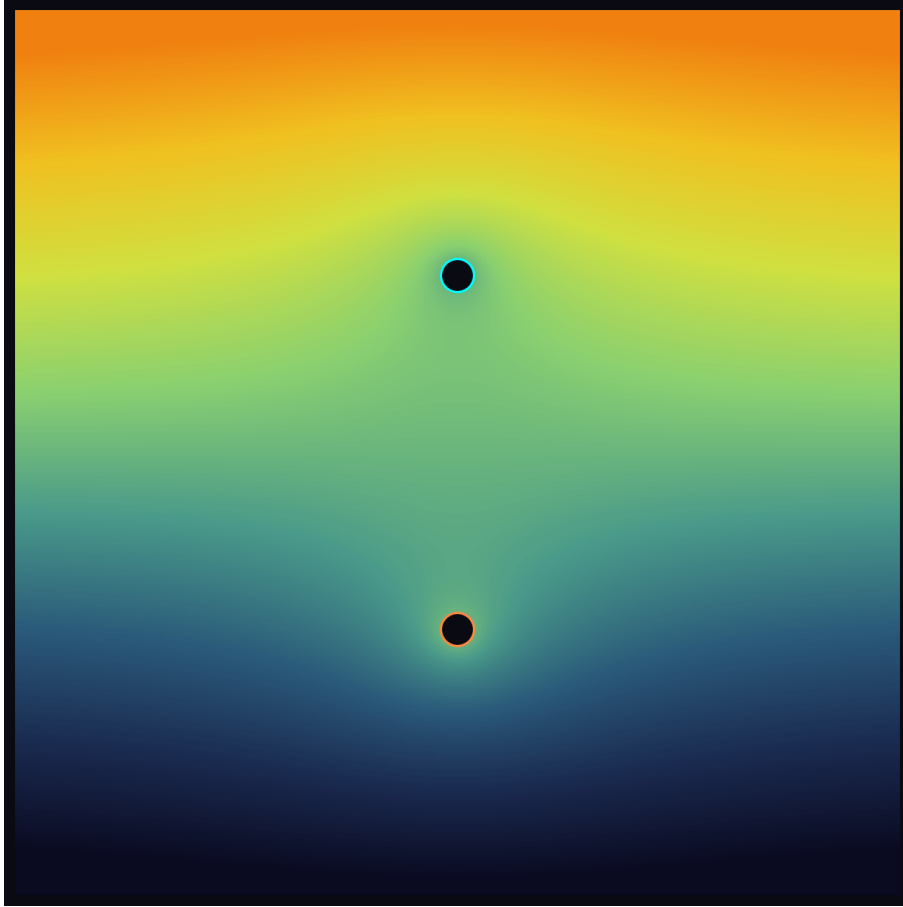


Figure 3: 2D gravitational potential near a planetary surface with stacked Gladosian portals. The planetary surface is along the bottom edge of the image, with ordinary gravity pointing down.

### 3 What's next?

Here are some ideas for fun things you might try next:

1. **Portals in 3D.** It's possible to solve poisson's equation in 3D; they involve the *spherical* harmonic functions (the same functions that give atomic orbitals their shape!). Though because you lose the conformal properties of flatland, the expressions for 3D gravitational potential are not so elegant; they mix fourier modes. In three dimensions, you remove two spheres from three dimensional space and glue the boundaries together — *it is very important that portals and their universes have the same dimension; otherwise you get sharp singularities*. Gladosian portals use *bispherical coordinates*, the analogue of bipolar coordinates in 3D.
2. **Relativity and spacetime.** The mathematics in this article is sort of a fun anachronistic clash — all of the gravity is purely newtonian, but it is happening in curved spaces. It would be interesting to do portals in a setting where gravity is curved spacetime. Then there are questions about time dilation and so on — I'll bet you can't cheat reality there either.
3. **Faraway Gladosian portals become Narnian portals.** I suspect there's a sense in which if you take the limit of a gladosian portal, pinning the point mass relative to one portal and sending the other portal to infinity, you get a narnian portal. That is, the portals are so far apart as to be in separate universes.

I think the math on this is a little tricky — intuitively, the infinitely many reflections of the portal get crowded into the center until there's only one reflection per portal. But those reflections are full of alternating  $\pm m$  masses and so you'd probably have to be careful about how you regularize while taking the limit.

But it would be interesting to prove.

4. **Portals and orbits.** If you fix a point mass in place near a pair of gladosian portals, both it and its mirror image seem to produce a gravitational field. What kinds of orbits do test masses make?

I think this is related to the (solvable!) restricted version of the three body problem that Euler solved, where two of the three masses are fixed in place. Here, the point mass and its reflection are fixed in space.

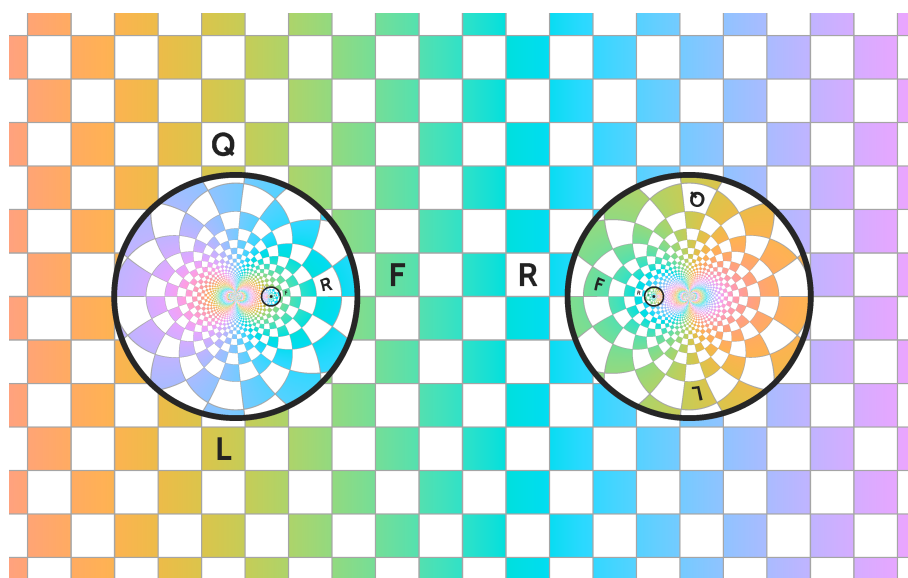


Figure 4: Each gladosian portal contains a picture of the whole infinite plane as seen from its counterpart portal's point of view. In this illustration, the colorful grid and the symmetry-free letters provide visual landmarks to show what connects to what. Note, for example, that the inside rim of each portal reflects the outside rim of the other portal.